Turbulent motions in the Atmosphere and Oceans

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Course description

The course will present the phenomena, theory, and modeling of turbulence in the Earth's oceans and atmosphere. The scope will range from centimeter to planetary scale motions. The regimes of turbulence will include homogeneous isotropic three dimensional turbulence, convection, boundary layer turbulence, internal waves, two dimensional turbulence, quasi-geostrophic turbulence, and planetary scale motions in the ocean and atmosphere. Prerequisites: the mathematics and physics required for admission to the graduate curriculum in the EAPS department, or consent of the instructor.

Course requirements

Class attendance and discussion, weekly homework assignments.

Reference texts

Andrews, Holton, and Leovy, "Middle atmosphere dynamics"
Frisch, "Turbulence: the legacy of Kolmogorov"
Lesieur, "Turbulence in Fluids", 3rd revised edition
McComb, "The physics of turbulence"
Saffman, "Vortex dynamics"
Salmon, "Lectures on geophysical fluid dynamics"
Tennekes and Lumley, "A first course in Turbulence"
Whitham, "Linear and nonlinear waves"

Chapter 5

Boundary layer turbulence

Turbulence in the ocean and atmosphere is strongly affected by the presence of boundaries. Boundaries impose severe modifications to the momentum and buoyancy budgets. At solid boundaries, the boundary condition that the fluid velocity is zero applies to both the mean velocity and to the fluctuations. Thus the turbulent fluxes of momentum must vanish. At the ocean free surface winds apply a stress that drives strongly turbulent motions. Finally, fluxes of heat, salt, and moisture at the boundaries can generate vigorous turbulent convection. Before discussing in detail the physics of planetary boundary layers in the ocean and atmosphere, it is useful to review some fundamental results that apply to stratified turbulence in general.

5.1 Turbulence in stratified fluids

In the previous lectures we studied the effect of stratification on oceanic macroturbulence, i.e. on turbulent motions with length and time scales large enough that rotation was important. In this limit motions have time to come in geostrophic balance with the density field and vertical velocities are strongly suppressed. The situation is quite different at the atmosphere and ocean boundaries, because surface fluxes continuously upset the gesotrophic balance and vertical motions are vigorous. In this lecture we first review the basics of stratified turbulence at scales where rotation is not important. Then we use the results to discuss boundary layer turbulence.

5.2 Mixing of stratified fluids

See Benoit Cushman-Roisin, section 11-1.

5.3 Turbulence in a stratified shear flow

Let us consider shear-driven turbulence at solid boundaries. At flluid boundaries, the condition that the fluid velocity is zero applies at every instant in time. Thus it applies to the mean velocity and the fluctuations separately,

$$\bar{\boldsymbol{u}} = 0, \qquad \boldsymbol{u}' = 0. \tag{5.1}$$

The fact that the fluctuations drop to zero at the wall has the particular implication that the Reynolds stress vanish,

$$-\overline{u_i u_j} = 0. \tag{5.2}$$

The only stress exerted directly on the wall is the viscous one. Away from the wall, instead, turbulence generates a Reynolds stress typically large compared to the viscous stress. Tritton (chapter 5, page 337) shows in Figure 21.12 the transition between a viscous stress and a turbulent stress in a turbulent boundary layer experiment (Schubauer, J. Appl. Physics, 1954). The total stress parallel to the wall does not change with distance from the wall, but there is an exchange of balance between the viscous and turbulent contributions.

To simplify the algebra let us consider a parallel irrotational flow over a flat boundary. Turbulence is generated because the no-slip condition $\bar{u} = 0$ at the boundary means that a shear layer results, and vorticity is introduced into the flow. Without loss of generality we can assume a constant background flow $\bar{u}_0$, which is independent of distance along the plate x and distance normal to the plate z. We also restrict the analysis to 2-dimensional flows, i.e. $\partial/\partial y = 0$, and assume that downstream evolution is slow. If L is a streamwise lengthscale, we are assuming that L is much larger than the viscous sublayer width across which the flow goes to zero $\delta/L << 1$, so that we can neglect variations in the streamwise direction compared to those in the vertical for averaged variables (i.e. $\partial/\partial x = 0$). Given these assumptions, the Reynolds averaged equations become,

$$\bar{w}\frac{d\bar{u}}{dz} = \frac{d}{dz}\left(\nu\frac{d\bar{u}}{dz} - \overline{w'u'}\right), \qquad \frac{d\bar{w}}{dz} = 0. \tag{5.3}$$

Because of the no normal flow through the boundary, we have $\bar{w} = w' = 0$ at $z = 0$, the bottom boundary. Then from eq. (5.3b) $\bar{w} = 0$ for all z. Then eq. (5.3a) becomes,

$$\frac{d}{dz}\left(\nu\frac{d\bar{u}}{dz} - \overline{w'u'}\right) = 0. \tag{5.4}$$

Hence if we have a stress τ given by,

$$\tau = \nu\frac{d\bar{u}}{dz} - \overline{w'u'} = \left(\nu\frac{d\bar{u}}{dz}\right)_{z=0}, \tag{5.5}$$

this stress is constant throughout the boundary layer. Near the boundary the stress is dominated by the viscous term. Away from the boundary we will have,

$$\tau = -\overline{w'u'}. \tag{5.6}$$

We can define a velocity scale from this surface stress

$$u_*^2 = \tau, \tag{5.7}$$

where u_* is the **friction velocity**. Away from the boundary eq. (5.6) implies that u_* is the turbulent velocity fluctuation magnitude.

Further reading: Tritton, chapter 21, 336–344 and Cushman-Roisan, section 11-3.

5.4 Convection

Convection is the process by which vertical motions modify the buoyancy distribution in a fluid. In the example considered above, the mixing of the upper ocean layer is caused by the mechanical action of the wind stress, and convection is said to be forced. Free convection arises when the only source of energy is of thermodynamic origin, such as an imposed heat flux. A common occurrence of free convection in geophysical fluids is the development of an unstable atmospheric boundary layer.

Kerry and Glenn showed that free convection occurs when the Rayleigh number Ra,

$$Ra = \frac{\Delta b h^3}{\nu \kappa_T}, \tag{5.8}$$

exceeds a critical value, which depends on the nature of the boundary conditions. For a fluid confined between two rigid plates and maintained at different tempertures at the two plates, the critical Rayleigh number is $Ra = 1708$. At values slightly over the threshold, convection organizes itself in parallel two dimensional rolls or in packed hexagonal cells. At higher values of the Rayleigh number, erratic time dependent motions develop, and convection appears much less organized.

Geophysical flows almost always fall in this last category, because of the large depths involved and the small values of molecular viscosity and diffusivity of air and water. In the atmospheric and oceanic boundary layer, where the Rayleigh number easily exceeds 10^{15}, convection is manifestly turbulent and viscosity/diffusivity play secondary roles. In this limit, the usntably-stratified part of the water column mixes to become essentially uniform. For fixed buoyancy boundary conditions, thin layers develop near the boundaries with thickness such that the local Rayleigh number is nearly critical. if the flux of buoyancy is fixed, these layers do not occur and the buoyancy gradient decreases to small values.

5.5 Ocean Mixed Layer Models

5.5.1 Bulk Mixed Layer Models: Price Weller and Pinkel Model

Price, Weller, and Pinkel (PWP) proposed a simplified boundary layer model for the upper ocean. The model is based on simple heuristic arguments and has proved quite accurate. The model adjusts the distributions of momentum and tracer properties, and in doing that it sets the mixed layer depth. The internal workings are rather simple. After adding the surface forcing, one applies three criteria for vertical stability (i.e. whether water should mix vertically, and whether the mixed layer should deepen). After that, it applies advection and diffusion (vertical advection and vertical diffusion) to the water column.

Static Stability Criterion

The first stability criterion, and the one that proves the most important in the model, is static stability. In fact, it accounts for about 80% of the "action". Quite simply put, one cannot have denser water overlying lighter water. This means that one must have $\partial_z \bar{b} \geq 0$. Thus one goes through the model domain (let "i" be the position index, with "i" increasing downward), one tests to make sure that,

$$\bar{b}_i \leq \bar{b}_{i+1}, \tag{5.9}$$

and where this is not the case, one then mixes all the cells above this depth (that is average them among themselves). In general, what one should really do is to just mix the two cells together, then start from the top of the model and do it again. What happens in practice, however, is since all the heat exchange (in particular cooling, which decreases buoyancy) takes place at the top of the model, one always finds that the effect of this instability is to mix all the way back to the top. So one may as well do it the first time. This scheme is equivalent to the convective overturning scheme described above, if one sets the diffusivity to infinity whenever there is static instability.

Bulk Richardson Number Stability Criterion

The second stability criterion is the bulk Richardson Number stability. This arises due to the fact that if the mixed layer gets going too fast (i.e. the wind stress is allowed to accelerate it to too great a speed), it tends to "stumble" over itself. What actually happens is that if there is too much velocity shear at the base of the mixed layer, it will tend to mix downward. This effect, determined by field and laboratory experiments is such that the mixed layer deepens if the bulk Richardson number goes below a critical value,

$$R_b = \frac{h\ \Delta\bar{b}}{|\Delta\bar{\boldsymbol{u}}|^2} \geq 0.65, \tag{5.10}$$

where h is the height (thickness) of the mixed layer, $\Delta\bar{b}$ is the buoyancy contrast between the mixed layer and the water below, and $\Delta\bar{\boldsymbol{u}}$ is the difference in horizontal velocity between the mixed layer and the underlying water. This effect tends to be important when the mixed layer becomes very thin, because a thin mixed layer becomes easily accelerated by wind stress, and the inverse quadratic nature of the dependence makes for a strong damping. The relative activity of this process is about 20% of the static instability.

Gradient Richardson Number Stability Criterion

The third stability criterion is based on the gradient Richardson number, and has the effect of stirring together layers where the velocity gradient becomes too great. One can think of this as the mixed layer "rubbing" against the water underneath it. This largely has the effect of blurring the transition between the mixed layer and the seasonal thermocline below, which would normally be rather sharp. Laboratory experiments indicate that there is a critical gradient Richardson number, below which stirring occurs,

$$R_g = \frac{\partial_z\bar{b}}{|\partial_z\bar{\boldsymbol{u}}|^2} \geq 0.25. \tag{5.11}$$

This turns out to be a not very vigorous process, but becomes a little more important in the absence of any explicit turbulent vertical diffusion. Notice that the gradient Richardson number introduced by Price, Weller, and Pinkel differs from the one used in KPP in that it does not include any parameterization for unresolved turbulent shear.

5.6 K-Profile Parameterization (KPP)

This boundary layer scheme due to Large, McWilliams and Doney (1994), is based on boundary layer theory and prescribes a profile of eddy diffusivities as a function of depth relative to the total oceanic boundary layer depth. The vertical profiles are based on the results from Prandtl boundary layer model and its corrections to include buoyancy forcing. The model is developed for the ocean. It accounts for both wind-stirring and convection. The main controlling parameter is $\sigma = z/h$, where z is the distance from the surface, and h is the depth of the boundary layer. Other controlling parameters are the surface fluxes of momentum and buoyancy, τ_0 and $\overline{w'b'}_0$, the Monin-Obukhov length-scale L_b, and the nondimensional ratio $\zeta = d/L_b$.

Non-local closure

All of the methods we have discussed thus far for parameterizing the turbulent fluxes of heat and momentum involve **local closures** - i.e. a relationship between the local values of the large scale gradient and the turbulent flux. However there can be situations in which a local relationship does not exist, because the local fluxes are generated by instabilities at a different location or time, and therefore are not related to the local mean gradients. In general it is difficult to deal with these situations, but there are some cases for which **non-local closures** can be derived. KPP incorporates a nonlocal closure. The vertical fluxes are parameterized as,

$$\overline{w'x'} = -K_x\left(\frac{\partial \bar{x}}{\partial z} - \gamma_x\right) \tag{5.12}$$

where γ_x is the nonlocal flux term. Nonlocal fluxes of scalars are important in convective boundary layers, i.e. when buoyant production is the dominant source of TKE. Then scalars are largely homogenized over the convective layer, but fluxes are still finite. γ_x is set to zero when there is no buoyant convection ($\overline{w'b'}_0 \le 0$), but is nonzero for scalars when $\overline{w'b'}_0 > 0$,

$$\gamma_x = C_s\frac{\overline{w'x'}_0}{w(\sigma)h} \tag{5.13}$$

Here $\overline{w'x'}_0$ is the surface tracer loss. The vertical velocity scale $w(\sigma)$ depends on position within the boundary layer (i.e. whether or not σh is greater than or smaller than the Monin-Obukhov lengthscale). In the convective limit ($h \gg L_b$) then,

$$\gamma_x = C^*\frac{\overline{w'x'}_0}{w^*h}, \tag{5.14}$$

where w^* is the convective velocity scale,

$$w^* \sim (\overline{w'b'}_0 h)^{1/3}, \tag{5.15}$$

$\overline{w'b'}_0$ is the surface flux, and $C^* = 10$.

K_x, the turbulent diffusivity, is given as a function of the depth of the turbulent boundary layer h (which is diagnosed from the mean density field), the turbulent velocity scale w, and a non-dimensional shape function $G(\sigma)$, where $\sigma = z/h$,

$$K_x(\sigma) = hw_x(\sigma)G(\sigma) \tag{5.16}$$

This is of a similar form to eq.(**??**), but now $w_x(\sigma)$ is determined diagnostically, and the turbulent lengthscale is a function of position within the boundary layer, and scales with h, the depth of the boundary layer. The shape function $G(\sigma)$ is assumed to be a cubic polynomial,

$$G(\sigma) = \sigma + a_2\sigma^2 + a_3\sigma^3, \tag{5.17}$$

with a_2, a_3 constants.

Substituting in eq.(5.12) we see that in the convective limit ($d\bar{x}/dz = 0$), the tracer flux is given by,

$$\overline{w'x'} = C^* \overline{w'x'}_0 G(\sigma) \tag{5.18}$$

so that the flux is proportional to the surface flux, scaled by a function of the position within the mixed layer.

Wind-stirred limit

Following boundary layer theory, the velocity profiles $w_x(\sigma)$ are given by,

$$w_x(\sigma) = \frac{\kappa u_*}{\phi(\zeta)} \tag{5.19}$$

where $\phi(\zeta)$ is the self-similar function introduced in the previous lecture to account for buoyancy effects on shear-driven turbulence. Let's consider the momentum budget. In the surface layer, $\sigma < 0.1$, where similarity theory applies we have,

$$\overline{w'u'} = u_*^2 = -h w_u(\sigma) G(\sigma) \frac{d\bar{u}}{dz} = -h\sigma \frac{\kappa u_*}{\phi(\zeta)} \frac{d\bar{u}}{dz}, \tag{5.20}$$

which gives the logarithmic layer profile,

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\kappa z} \phi(\zeta). \tag{5.21}$$

Then in the surface layer the diffusivity/viscosity is given by,

$$K_x(\sigma) = \frac{\kappa u_* z}{\phi(z)}. \tag{5.22}$$

Boundary layer depth

One of the most important aspects of KPP is the dependence of parameters on the total boundary layer depth. This adds another non-local aspect to the parameterization. The boundary layer depth is determined as the location where the bulk Richardson number exceeds some critical value (0.3), with the Richardson number $Ri(z)$ defined as

$$Ri(z) = \frac{[\bar{b}(0) - \bar{b}(z)]\ z}{|\bar{\boldsymbol{u}}(0) - \bar{\boldsymbol{u}}(z)|^2 + \overline{|\boldsymbol{u}_t(z)|^2}}. \tag{5.23}$$

$\bar{\boldsymbol{u}}(0)$ and $\bar{b}(0)$ are the near surface resolved velocity and buoyancy (averaged over the surface layer, the top 10% of the boundary layer). $\boldsymbol{u}_t/z$ is the turbulent shear, which has to be parameterized. This $\boldsymbol{u}_t$ term is very important, because it reduces the Ri and hence extends the depth of the boundary layer over which enhanced diffusivities are applied into the stably stratified region. If $\boldsymbol{u}_t$ is set to zero, then KPP does not reproduce penetrative convection. The value of $\boldsymbol{u}_t$ depends on the ratio of the reverse (penetrative) buoyancy flux at the base of the convective layer to the forcing $\overline{w'b'}_0$, which is known empirically to be 0.2.

For stable forcing $\overline{w'b'}_0 < 0$, so that $L_b > 0$, upper limits are imposed on the boundary layer depth,

$$h < L_b, \qquad \text{and} \qquad h < h_E = 0.7u^*/f, \tag{5.24}$$

where h_E is the Ekman layer depth.

Momentum versus tracers

Momentum and tracer fluxes are parameterized in a similar fashion in KPP except for the following,

- $\phi > \phi_x$ when $L_b < 0$ (i.e. convective forcing), so that tracers are mixed more efficiently than momentum.
- $\gamma_m = 0$ - no counter-gradient fluxes of momentum in convective scenarios.